Name:____ Date:__________________________

Algebra 2 CC: Multiplying Radicals

Recall that if a and b are non-negative numbers, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. Therefore, by the symmetric property of equality, we can say that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$. Recall also that for any positive number a, $\sqrt{a} \cdot \sqrt{a} = \sqrt{a^2} = a$. We can use these rules to multiply radicals.

For example:

$$\sqrt{4} \cdot \sqrt{25} = \sqrt{100} = 10$$

$$\sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2$$

$$\sqrt{8} \cdot \sqrt{2} = (\sqrt{4} \cdot \sqrt{2}) \cdot \sqrt{2} = 2(\sqrt{2} \cdot \sqrt{2}) = 2(2) = 4$$

$$\sqrt{6a^3} \cdot \sqrt{18a} = \sqrt{108a^4} = \sqrt{36a^4} \cdot \sqrt{3} = 6a^2\sqrt{3} \quad (a \geq 0)$$

Note: $\sqrt{-2} \times \sqrt{-8} \neq \sqrt{16}$ because $\sqrt{-2}$ and $\sqrt{-8}$ are not real numbers.

The distributive property for multiplication over addition or subtraction is true for all real numbers. Therefore, we can apply it to irrational numbers that contain radicals.

For example:

$$\sqrt{3}(2 + \sqrt{3}) = \sqrt{3}(2) + \sqrt{3}(\sqrt{3}) = 2\sqrt{3} + 3$$

$$\begin{aligned}(2 + \sqrt{5})(1 + \sqrt{5}) &= 2(1 + \sqrt{5}) + \sqrt{5}(1 + \sqrt{5}) \\ &= 2 + 2\sqrt{5} + \sqrt{5} + 5 = 7 + 3\sqrt{5}\end{aligned}$$

$$\begin{aligned}(3 + \sqrt{2})(3 - \sqrt{2}) &= 3(3 - \sqrt{2}) + \sqrt{2}(3 - \sqrt{2}) \\ &= 9 - 3\sqrt{2} + 3\sqrt{2} - 2 = 7\end{aligned}$$

Express each of the following products in simplest form:

1. $\sqrt{5}(\sqrt{10})$

2. $(3 + \sqrt{6a})(1 + \sqrt{2a})$

3. $(\sqrt{5} + \sqrt{7})(\sqrt{5} - \sqrt{7})$

HW:

In 3–41, express each product in simplest form. Variables in the radicand with an even index are non-negative.

3. $\sqrt{2} \cdot \sqrt{8}$
4. $\sqrt{5} \cdot \sqrt{45}$
5. $\sqrt{3} \cdot \sqrt{27}$
6. $\sqrt{8} \cdot \sqrt{12}$
7. $-\sqrt{10} \cdot \sqrt{18}$
8. $3\sqrt{2} \cdot \sqrt{10}$
9. $\sqrt{\frac{1}{3}} \cdot \sqrt{24}$
10. $\sqrt{21} \cdot \sqrt{\frac{4}{3}}$
11. $8\sqrt{6} \cdot \sqrt{\frac{5}{12}}$
12. $(\sqrt{12})^2$
13. $(3\sqrt{3})^2$
14. $(-2\sqrt{5})^2$
15. $\sqrt{x^3} \cdot \sqrt{4x}$
16. $2\sqrt{ab} \cdot 2\sqrt{ab^2}$
17. $\sqrt{5y} \cdot \sqrt{4y^3}$
18. $\sqrt{x^5y^3} \cdot \sqrt{3xy}$
19. $7\sqrt{a} \cdot 5\sqrt{\frac{a}{9}}$
20. $\sqrt{\frac{x}{2}} \cdot \sqrt{\frac{x^2}{2}}$
21. $\sqrt{\frac{a}{3}} \cdot \sqrt{\frac{a^2}{4}}$
22. $\sqrt[3]{2} \cdot \sqrt[3]{4}$
23. $\sqrt[3]{15a^2} \cdot \sqrt[3]{9a^4}$
24. $\sqrt[4]{27} \cdot \sqrt[4]{3}$
25. $\sqrt{2}\,(2 + \sqrt{2})$
26. $\sqrt{5}(1 - \sqrt{10})$
27. $\sqrt{8}(6 + \sqrt{2})$
28. $\sqrt{5a}(\sqrt{5a} - 3)$
29. $\sqrt{12xy^3}(\sqrt{3xy} + 3)$
30. $(1 + \sqrt{5})(3 - \sqrt{5})$
31. $(9 + \sqrt{2b})(1 + \sqrt{2b})$
32. $(7 + \sqrt{5y})(3 - \sqrt{5y})$
33. $(7 + \sqrt{5b})(7 - \sqrt{5b})$
34. $(x - \sqrt[4]{3y})(2x - \sqrt[4]{3y})$
35. $(\sqrt{6} + 6)(\sqrt{6} - 7)$
36. $(\sqrt{6} + 6c)(\sqrt{6} - 6c)$
37. $(a + \sqrt{b})(a - \sqrt{b})$
38. $(1 - \sqrt{3})^2$
39. $(3 + \sqrt{5ab^3})^2$
40. $(1 - \sqrt{7})(1 + \sqrt{7})(1 + \sqrt{7})$
41. $(2 - \sqrt{5})(2 + \sqrt{5})^2$

Applying Skills

42. The length of a side of a square is $48\sqrt{2}$ meters. Express the area of the square in simplest form.

43. The dimensions of a rectangle are $12\sqrt{2}$ feet by $\sqrt{50}$ feet. Express the area of the rectangle in simplest form.